the mylar/foam surface are planned to obtain detailed hot-wire, pitot, and surface displacement measurements at numerous survey stations along the compliant wall. Other promising surfaces will also be examined experimentally as an integral part of a continuing theoretical, experimental, and materials program to understand the drag reducing mechanism and to analyze the frequency, amplitude, and vibrational behavior of low-modulus materials. The ultimate goal is to develop practical and durable compliant surfaces optimized for maximum drag reduction, with applications to the fuselage of the next generation of subsonic/supersonic transports and possible application as a retrofit to existing transports. It should be noted that the present data are the first compliant wall test results at dynamic pressure corresponding to flight conditions of subsonic transport aircraft.

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Supersonic Flutter of Parallel Flat Plates Connected by an Elastic Medium

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Introduction

THE configuration to be analyzed is shown in Fig. 1 and consists of an upper isotropic plate over which there is a supersonic air flow, separated by a linear elastic medium from a lower isotropic plate which may have different material properties and in-plane loadings from the upper plate. All plate edges are assumed to be simply-supported.

The flutter behavior of this configuration was investigated in Ref. 1 using a "static" supersonic strip theory in a two-mode Galerkin approach and sufficient graphical results were presented to indicate that the elastic connecting medium could have significant effects which depended also on the values of the in-plane loads.

The purpose of this Note is to present analyses based on a similar approach, and also on the use of a finite difference procedure, but using linear piston theory, and from which the influences of the various parameters can more readily be identified. Comparisons are made with the results of Ref. 1 and, for the case of the lower plate rigid, with Ref. 2.

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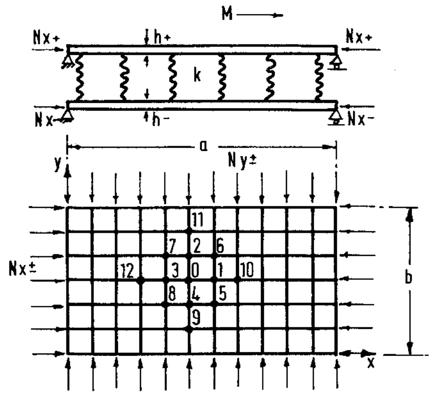

Fig. 1 Configuration and coordinate system showing finite difference mesh.

Governing Equations

The governing differential equations and boundary conditions are[1]

$$D_+\nabla^4 w_+ + N_{x+}\cdot(\partial^2 w_+/\partial x^2) + N_{y+}\cdot(\partial^2 w_+/\partial y^2)$$
$$+\delta_+ h_+(\partial^2 w_+/\partial t^2) + k(w_+ - w_-) = L_{(w_+)} \qquad (1)$$

$$D_-\nabla^4 w_- + N_{x-}(\partial^2 w_-/\partial x^2) + N_{y-}(\partial^2 w - /\partial y^2)$$
$$+\delta_- h_-(\partial^2 w_-/\partial t^2) + k(w_- - w_+) = 0 \qquad (2)$$

$$w\pm(x,o,t) = w\pm(x,b,t) = w\pm(o,y,t) = w\pm(a,y,t) = 0$$
$$(\partial^2 w\pm/\partial y^2)(x,o,t) = (\partial^2 w\pm/\partial y^2)(x,b,t)$$
$$= (\partial^2 w\pm/\partial x^2)(o,y,t) = (\partial^2 w\pm/\partial x^2)(a,y,t) = 0 \qquad (3)$$

The subscripts $\pm$ refer to upper and lower plate, respectively, and all other notation is standard and as in Ref. 1.

In Eq. (1) the lateral aerodynamic pressure L is represented by the following simple expression from linear piston theory

$$L = -[P_I \dot{w}_+ + (P_2/a) w'_+] \qquad (4)$$

where the prime and dot denotes a differentiation with respect to (x/a) and time (t), respectively. Clearly $P_I = 0$ when aerodynamic damping is neglected.

Structural damping has not been considered in general. However hysteretic damping in the connecting medium can be represented by $k_g = k(1+ig)$.

Galerkin Procedure

If assumed functions $X\pm$ and $Y\pm$ are substituted into Eqs. (1) and (2), then these equations are not, in general, satisfied identically, and the Galerkin method may be invoked. It can easily be shown that for simply supported edges and the assumption of trigonometric functions, the chordwise modes (in x) and the spanwise modes (in y) must be similar for the upper and lower plates for a coupled plate solution to be possible.

Based on the two modes

$$w\pm = C_{mn\pm}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$
$$+ C_{rn\pm}\sin\left(\frac{r\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (5)$$

the following flutter determinant is obtained when $(r-m)$ is odd.

$$\begin{vmatrix} K_m, & -4rm\lambda[\pi^4(r^2-m^2)]^{-1} \\ +4rm\lambda[\pi^4(r^2-m^2)]^{-1} & K_r \end{vmatrix} = 0 \quad (6)$$

where

$$K_m = m^4 - m^2 A_+ - B_+ + \overset{\Omega}{S}_+ - S_+ S_- [m^4 - m^2 A_- - B_- + S_-]^{-1} \quad (7)$$

and

$$\begin{aligned} A_\pm &= R_{x\pm} - 2n^2(a/b)^2; \\ B_\pm &= \Delta_\pm + n^2 R_{y\pm}(a/b)^2 - n^4(a/b)^4; \\ \Delta_\pm &= \Omega^2 a^4 \delta_\pm h_\pm / \pi^4 D_\pm; \ \delta = \text{mass density} \\ S_\pm &= k_g a^4/\pi^4 D_\pm \\ R_{x\pm} &= N_{x\pm} a^2/\pi^2 D_\pm \ \text{etc.} \\ \overset{\Omega}{S}_+ &= S_+ + P_1 i\Omega a^4/\pi^4 D_+ \\ \lambda &= P_2 a^3/D_+; \ \Omega = \text{circular frequency} \end{aligned} \quad (8)$$

All other notation is standard as in Ref. 1. If $n=1$, $m=1$, $r=2$, and $P_1 = g = 0$ Eq. (6) reduces to Eq. (5) of Ref. 1.

Finite Difference Procedure

If both upper and lower plates in Fig. 1 are considered to be divided into an identical square mesh of stations as shown, the various plate deflections at these stations can be used as eigenvectors in the title problem when the governing differential Eqs. (1) and (2) are written in finite difference form.

Using Ref. 3 the equation for a general internal station on the upper plate remote from an edge (e.g., station w_o on Fig. 1) are

$$\begin{aligned} &D_+[20w_o - 8\sum_1^4 w_s + 2\sum_5^8 w_s + \sum_9^{12} w_s] \\ &+ N_{x+}d^2[w_1 + w_3 - 2w_o] \\ &+ N_{y+}d^2[w_2 + w_4 - 2w_o] + \delta_+ h_+ d^4\Omega^2 \\ &+ k_g d^4[w_o - v_o] = d^4 L_{(x,y,t)} \\ &= -d^4[P_1 i\Omega w_o + (P_2/2d)(w_1 - w_3)] \end{aligned} \quad (9)$$

where w, v represent the displacements of the upper and lower plates respectively, and the corresponding equation for the lower plate in terms of the various v_s and w_o is similar to Eq. (9) but with the right-hand side zero. Similar equations can be derived for all stations, as indicated in Refs. 3 and 4 and with due consideration of the influence of any form of edge boundary condition.

Flutter of Coupled Plates

Expansion of the full flutter determinant in Eq. (6) and separation into real and imaginary parts yields two equations which yield, respectively, expressions for the flutter frequency parameter Δ_f and flutter speed parameter $\bar{\lambda}$. For identical plates loaded identically, and with $g=0$ these expressions become

$$[F_m + F_r - 2\Delta_f][1 - S^2(F_m - \Delta_f)^{-1}(F_r - \Delta_f)^{-1}] = 0 \quad (10)$$

$$\bar{\lambda}^2 = \bar{P}_1^2 + [(F_m - \Delta_f) - S^2(F_m - \Delta_f)^{-1}]^2 \quad (11)$$

where

$$\begin{aligned} F_m &= m^4 - m^2 A - n^2 R_y (a/b)^2 + n^4 (a/b)^4 + S \\ \bar{\lambda} &= 4rm\lambda[\pi^4(r^2-m^2)]^{-1} \\ \bar{P}_1 &= P_1 \Omega a^4/\pi^4 D \end{aligned} \quad (12)$$

and for linear piston theory

$$P_1 = \rho c; \quad P_2 = \rho U^2/M; \quad c = \text{speed of sound} \quad (13)$$

It should be noted that $\bar{P}_1^2$ in Eq. (11) can be written as

$$\bar{P}_1^2 = Q\Delta_f \lambda/\pi^4 \equiv g_A^2 \Delta_f \quad (14)$$

where Q is the aerodynamic damping parameter $(\rho a/\delta h M)$ and g_A is the aerodynamic damping coefficient of Ref. 2.

It can be shown that the coupled in-vacuo frequency parameter Δ_{mn} from Eq. (6) for this case is

$$\Delta_{mn} = [m^2 + n^2(a/b)^2]^2 - m^2 R_x - n^2 R_y (a/b)^2 + \begin{Bmatrix} 2S \\ \text{or} \\ 0 \end{Bmatrix} \quad (15)$$

and the amplitudes of motion of the two plates are identical and respectively out of phase (term $2S$ retained) or in-phase $(S=0)$.

Equation (10) gives three possible values for the flutter frequency parameter, viz.

$$\Delta_{f\binom{+}{-}} = (F_m + F_r)/2 \pm [(F_r - F_m)^2/4 + S^2]^{1/2} \quad (16)$$

or

$$\Delta_{f(o)} = (F_m + F_r)/2 \quad (17)$$

and if these are substituted into Eqs. (11) one obtains

$$\begin{aligned} \bar{\lambda}^2 &= g_A^2\Delta_f + [f - S^2 f^{-1}]^2 \quad &&, \text{for } \Delta_{f(o)}; S \neq 0 \quad (18) \\ &= g_A^2\Delta_f + [2f]^2 &&, \text{for } \Delta_{f\binom{+}{-}}; S \neq 0 \quad (19) \\ &= g_A^2\Delta_f + f^2 &&, \text{for } \Delta_{f(o)}; S = 0 \quad (20) \end{aligned}$$

and $f = (F_r - F_m)/2 \equiv (\Delta_{rn} - \Delta_{mn})/2$, where Δ_{rn} and Δ_{mn} are the uncoupled modal frequency parameters for a single panel with $S=0$.

The term $[f - S^2 f^{-1}]$ in Eq. (18) can be shown to be zero when $(\Delta_{rn} - \Delta_{mn})$ is zero; and using Eq. (15) this corresponds to

$$F_r - F_m = \pm 2S \text{ or } 0 \quad (21)$$

Thus, if $a/b = n = 1$; $r=2$, $m=1$; $S=20$, Eq. (21) is satisfied when $R_x = 61/3$, 7, $-19/3$ and clearly, a zero flutter speed can result when aerodynamic damping is neglected, even with tensile loads acting (R_x negative).

Equations (18-20) show that the parameter $\bar{\lambda}$ depends mainly on the difference in the uncoupled (or coupled) modal frequency parameters, and, through $g_A^2\Delta_f$, on their sum. Tensile values of R_y raise the values of Δ_{mn} and Δ_{rn} and help to stabilize the flutter through the aerodynamic damping term $g_A^2\Delta_f$, even though the term f in Eqs. (18-20) is independent of R_y. However, it is believed that this effect is generally small. Similar results have been shown previously for flat plates and cylindrical shells. Results obtained from Eqs. (18-20), with $g_A = 0$, are shown plotted on Figs. 2 and 3 for a two-dimensional $(a/b=0)$ and square $(a/b=1)$ configuration, respectively.

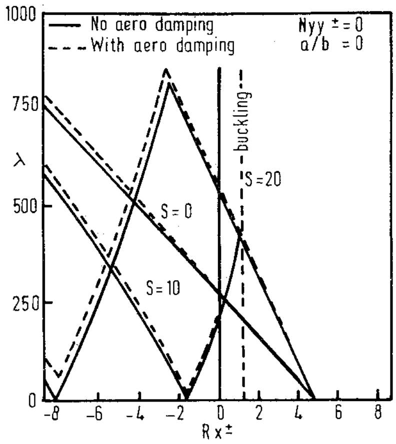

Fig. 2 Flutter boundary for $R_{x+} = R_{x-} = R_x$.

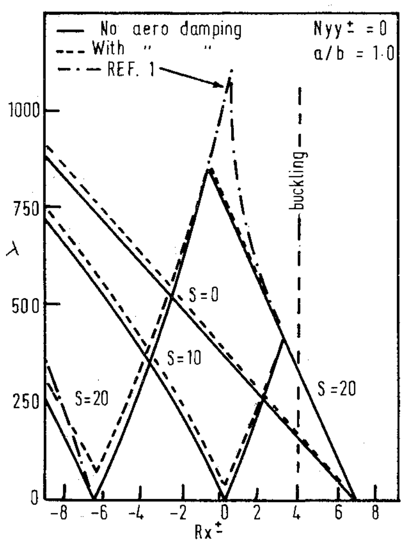

Fig. 3 Flutter boundary for $R_{x+} = R_{x-} = R_x$.

Figure 3 may be compared directly with Fig. 3 of Ref. 1 for $S=20$ which is superimposed. There are slight differences in the region of the peak values of λ which are as yet unexplained. According to the present analysis, the curves for a single panel ($S=0$) are given by Eq. (20) whereas for coupled panels Eq. (18) gives the region of the curves to the left of the peak, and Eq. (19) gives the region to the right. When $g_A=0$ λ is linear in R_x from Eqs. (19) and (20) and the slope of the line from Eq. (19) is twice that from Eq. (20).

From both Figs 2 and 3, the influence of S is quite marked, and it is seen that as S increases from 0 to 10 to 20, the flutter is progressively more critical for large tensile values of R_x. These results amplify those presented in Ref. 1 where, in addition, other combinations of in-plane loadings were examined.

No results are presented for the case of $g \neq 0$, but it is easily shown that if $g \neq 0$ and $g_A \to 0$ completely different results are obtained for Δ_f compared to Eqs. (10, 16, and 17) and, consequently, different values of λ are obtained. Similar analyses were presented in Ref. 5 for the case of $S=0$.

Flutter of Uncoupled Single Plates ($S=0$)

If it is assumed that the lower plate of Fig. 1 is infinitely rigid ($D_- = \infty$) the Eqs. (6-11) are correspondingly simplified, and Eqs. (10) and (11) yield, when $g=0$,

$$\Delta_f = [\Delta_m + \Delta_r]/2 \tag{22}$$

$$\bar{\lambda}^2 = g_A^2 \Delta_f + [(\Delta_r - \Delta_m)/2]^2 \tag{23}$$

If $n=1, r=2, m=1$,

$$\bar{\lambda}^2 = (8/3\pi^4)^2 \lambda^2 = g_A^2 [17 - 5\{R_x - 2(a/b)^2\} + 2S - 2R_y(a/b)^2 + 2(a/b)^4]^2 + [15 - 3\{R_x - 2(a/b)^2\}]^{2/4} \tag{24}$$

For the case when $R_x = R_y = a/b = 0$, it can readily be shown that only when $S \leq 100$ do reasonable results arise as compared with Fig. 9 of Ref. 2. Likewise when $R_x = R_y = S = 0$ the results obtained when compared with Fig. 8 of Ref. 2 are only satisfactory for $a/b \leq 1$.

In both these cases it is clear that the assumption of standing wave behavior, with the assumed mode numbers $r=2$, $m=1$, has proved inadequate at higher values of S and a/b when travelling wave type motions are more likely. Because of the form of Eq. (24) it can also be deduced that high values of g_A (e.g., $g_A > 20$) will also magnify the effects of higher values of S and a/b and tend to make Eq. (24) less applicable.

Flutter of Single Plates on Elastic Foundation ($S \neq 0$)

To illustrate the application of the finite difference formulation to the stated flutter problem of a square plate, the simple mesh shown in Fig. 4 is applied with only two unknown displacements w_A, w_B. The following flutter determinant is obtained:

$$\begin{vmatrix} H & J_1 \\ J_2 & H \end{vmatrix} = 0 \tag{25}$$

where

$$H = 10 - 2\bar{N}_x - \bar{N}_y - \bar{\Omega} + i\rho U \Omega d^4/MD$$

$$J_1 = -6 + \bar{N}_x + \rho U^2 d^3/2MD$$

$$J_2 = -6 + \bar{N}_x - \rho U^2 d^3/2MD$$

$$\bar{\Omega} = \Omega^2 \delta h d^4/d; \quad \bar{N}_x = N_x d^2/D \text{ etc.}$$

If aerodynamic damping is subsequently neglected, the following simple flutter criterion is obtained in the notation of Ref. 1:

$$\lambda \simeq 60[5.4 - R_x] \; ; \; R_x < 3.6 \tag{26}$$

The agreement of this result with that shown in Fig. 3 for $S=0$ is quite encouraging.

The corresponding criterion for a two-dimensional plate ($a/b=0$), which may be compared with Fig. 2, is

$$\lambda = 60[3.6 - R_x] \; ; \; R_x < 0.9 \tag{27}$$

Both Eqs. (26) and (27) should be compared with the general equation found from the Galerkin method, viz.

$$\lambda \simeq 55[5 + 2(a/b)^2 - R_x] \; ; \; R_x < [1 + (a/b)^2]^2 \tag{28}$$

and it will be seen that agreement is reasonable for tensile (negative) values of R_x ($-10 < R_x < 0$). In fact, none of the Eqs. (26), (27), or (28) is valid for compressive (positive) values of R_x beyond the buckling values indicated in those equations when $R_y=0$.

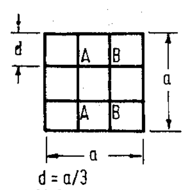

Fig. 4 Mesh configuration for square plate.

Conclusions

Analyses for the supersonic flutter of a system of elastically connected parallel plates have been made and general expressions derived for the appropriate nondimensional parameters as a function of geometry, stiffnesses, and in-plane loadings. The influence of the elastic connecting medium is critical in determining the severity of the flutter and it is also found that flutter may be more critical for large tensile values of the midplane (chordwise) stress resultant. The finite difference formulation gives most encouraging results from quite simple analyses and deserves further development particularly for panels with irregular planform contours.

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Nonlinear Equations for Shallow Sandwich Shells with Orthotropic Cores

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Nomenclature

B_i	$= E_i t_i/(1-\mu^2)$
C	= thickness of the core
C_1	$= (1-\mu^2)(B_1+B_2)$
D_i	$= E_i t_i^3/12(1-\mu^2)$
E_i	= elastic modulus of the ith facing
G_{xz}, G_{yz}	= shear moduli of the core in the x and y directions, respectively
h	$= C+(t_1+t_2)/2$
k	$= CB_1B_2/G_{xz}(B_1+B_2)$
k_1	$= G_{xz}/G_{yz}$
$\bar{k}$	$= h^2B_1B_2/(B_1+B_2)$
N_u^*, N_L^*	= upper and lower critical loads
R_1, R_2	= principal radii of curvature
t_i	= thickness of the ith facing
w	= deflection of sandwich shell
w_0	= undetermined coefficient

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I. Introduction

NONLINEAR equations for a shallow unsymmetrical sandwich shell of double curvature were obtained by Fulton.[1] However, in his analysis he treated the core to be isotropic, while in the actual constructions the cores are usually orthotropic. The purpose of this Note is to make a generalization of Fulton's equations[1] by including the effect of core orthotropy. The assumptions made in this Note with regard to faces and core are the same as those in Fulton's paper except the core is considered as orthotropic in this Note. As an example a square cylindrical shell section loaded in the longitudinal direction is considered, and the effects of core orthotropy upon the lower and upper critical loads are discussed.

II. Governing Equations

The same analytical procedure used in Ref. 1 can be used in this derivation. However, because of the presence of orthotropic core, the derivation becomes much more complex. After a rather lengthy process of substituion and differentiation, the following nonlinear eqations for shallow unsymmetrical sandwich shell of double curvature with orthotropic core are obtained[2]:

$$\nabla^4 F+(1-\mu^2)(B_1+B_2)\left[\frac{1}{R_1}\frac{\partial^2 w}{\partial y^2}+\frac{1}{R_2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left[\frac{\partial^2 w}{\partial x\partial y}\right]^2\right]=0 \tag{1}$$

$$\left[k\left[\frac{\partial^2}{\partial y^2}+k_1\frac{\partial^2}{\partial x^2}\right]-\frac{2}{(1-\mu)}\right]\times\left\{\left[2-\frac{(1-\mu)}{2}k(k_1+1)\nabla^2-(1+\mu)k\left(k_1\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)\right]\phi-2\nabla^2 w\right\}=k(k_1-1)\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-\frac{(1-\mu)}{2}k\left(k_1\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\nabla^2-(1+\mu)k(k_1-1)\frac{\partial^4}{\partial x^2\partial y^2}\right]\phi \tag{2}$$

$$(D_1+D_2)\nabla^4 w-\left[\frac{1}{R_1}+\frac{\partial^2 w}{\partial x^2}\right]\frac{\partial^2 F}{\partial y^2}-\left[\frac{1}{R_2}+\frac{\partial^2 w}{\partial y^2}\right]\frac{\partial^2 F}{\partial x^2}+2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}+\bar{k}\nabla^2\phi-q=0 \tag{3}$$

where

$$k_1=G_{xz}/G_{yz} \tag{4}$$

G_{xz} and G_{yz} are the shear moduli of the core in the x and y directions, respectively, and the rest of the notations are defined in Ref. 1. It can be readily shown that when $k=1$, i.e., for the case of isotropic core, Eqs. (1-3) reduce to Fulton's equations [Ref. 1, Eqs. (26,30, and 32)] as they should be.

III. Example

As an example, the equations derived herein are used to obtain the critical load and the snap-through load of a simply supported square shallow barrel shell panel with orthotropic core (Fig. 1). In this case, $a=b$, $R_1=\infty$, $R_2=R$, and N^* is the